Can modified gravity explain accelerated cosmic expansion?

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Abstract

We show that the recently suggested explanations of cosmic acceleration by the modification of gravity at small curvature suffer violent instabilities and strongly disagree with the known properties of gravitational interactions.

It seems established that at the present epoch the universe expands with acceleration. It follows directly from the observation of high red-shift supernovae [1] and indirectly from the measurements of angular fluctuations of cosmic microwave background fluctuations (CMBR) [2]. The latter shows that the total mass/energy density of the universe is very close to the critical one ($\Omega = 1$), while the observations of the universe large scale structure indicate that normal gravitating (but invisible) matter can contribute only 30% into the total one. Thus one concludes that the remaining 70% is some mysterious agent that creates the cosmological acceleration. The simplest suggestion is that the source of this acceleration is the vacuum energy (cosmological constant). However such an explanation immediately meets two serious problems. First, vacuum energy remains constant in the course of cosmic expansion (modulo possible phase transitions which could discontinuously change the value of ρ_{vac} by a very large amount). On the other hand, "normal" cosmological energy density scales with time as $\rho_m \sim m_{Pl}^2/t^2$ and it is quite strange coincidence that ρ_{vac} and ρ_m are of the same order of magnitude just today. Second and much more serious, there are known contributions into vacuum energy which are 50-100 orders of magnitude larger than the cosmologically allowed value. At the present day there is no workable scenario to solve the second problem, while the first one may be solved if instead of a constant ρ_{vac} a new massless or very light field is introduced which evolves in the course of the expansion in such a way that its energy density today more or less naturally approaches the energy density of normal matter [3].

Still such field remains mysterious. It would be in much better shape if it simultaneously solved the problem of almost complete elimination of gigantic contributions into vacuum energy by some adjustment mechanism but, alas, it does not happen or, better to say, it is unknown how it can be realized. To avoid an introduction of additional fields (plurality should not be posited without necessity", W. Ockham) it was suggested recently that gravity itself, if properly modified, could create cosmological acceleration [4]. To this end the model with the following action was considered:

$$\mathcal{A} = \int d^4x \sqrt{-g}\left[f(R) + \mathcal{L}_M\right] , \tag{1}$$

where R is the scalar curvature and $\mathcal{L}_M$ is the matter Lagrangian. In the usual Einstein gravity the function $f(R)$ has the form $f(R) = Rm_{Pl}^2/16\pi$ where m_{Pl} is the Planck mass ($\simeq 10^{19}$GeV). In ref. [4] a power law modification $f(R) \sim R^n$ with $n = -1, 3/2$ was considered.

A detailed examination of the $n = -1$ case was performed in the recent paper [5]. The action was taken in the form:

$$\mathcal{A} = \frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g} \left(R - \frac{\mu^4}{R} \right) - \int d^4x \sqrt{-g}\, \mathcal{L}_M \,, \tag{2}$$

and the corresponding equation of motion reads:

$$\left(1 + \frac{\mu^4}{R^2}\right) R_{\alpha\beta} - \frac{1}{2}\left(1 - \frac{\mu^4}{R^2}\right) R g_{\alpha\beta} + \mu^4 g_{\alpha\beta} \nabla_\nu \nabla^\nu \left(\frac{1}{R^2}\right) - \mu^4 \nabla_{(\alpha} \nabla_{\beta)} \left(\frac{1}{R^2}\right) = \frac{8\pi\, T^M_{\alpha\beta}}{m_{Pl}^2} \,, \tag{3}$$

where $T^M_{\mu\nu}$ is the matter energy-momentum tensor. As argued in ref. [5], with the parameter μ chosen of the order of the inverse universe age, $\mu^{-1} \sim 10^{18}\,\text{sec} \sim (10^{-33}\text{eV})^{-1}$, this equation, applied to cosmology, describes the universe acceleration in the present epoch, while the additional terms were not essential at earlier times.

However an addition of non-linear in curvature terms into the action integral is not an innocent step. It gives rise to higher order equations of motion which possess quite unpleasant pathological behavior. They may break unitarity and lead to ghosts or tachyons. And if breaking of the usual gravity at high curvatures may be tolerated - who knows what happens there - but breaking of gravity at low curvature in weak field regime immediately leads to contradiction with well established facts. To demonstrate that let us consider equation of motion for curvature scalar. It can be obtained from eq. (3) by taking trace over α and β:

$$D^2 R - 3\, \frac{(D_\alpha R)\,(D^\alpha R)}{R} + \frac{R^4}{6\mu^4} - \frac{R^2}{2} = -\frac{T\, R^3}{6\mu^4} \,, \tag{4}$$

where D is the covariant derivative and $T = 8\pi T^\nu_\nu / m_{Pl}^2 > 0$.

Let us apply this equation for the gravitational field of a normal celestial body, as e.g. the Earth or the Sun or any other smaller gravitating object with weak gravity. In this case the metric can be approximately taken as a flat one, so $D^2 = \partial_t^2 - \Delta$ and $(D_\alpha R)(D^\alpha R) = (\partial_t R)^2 - (\partial_j R)^2$. We will look for a perturbative solution inside some spatially finite matter distribution. In the lowest order the curvature is algebraically expressed through the trace of the energy-momentum tensor of matter, $R_0 = -T$. This is the standard result of General Relativity (GR). Outside matter the solution is $R_0 = 0$ but now this is not exact but true only in the lowest order approximation. One can check both numerically and analytically that stationary solution outside matter sphere is very quickly vanishing if $\mu^4 > 0$. So at least in this case the curvature scalar behaves similarly to the usual one. This is in agreement with the analysis of stationary solutions of ref. [6] where it is argued that the modified gravity theories agree with the Newtonian limit of the standard gravity for sufficiently small μ.[1]

However for negative μ^4 the solution is rising outside the matter sphere and strongly deviates from the standard General Relativity. Even for $\mu^4 > 0$, the curvature scalar, R, outside the source would be non-vanishing and proportional to the curvature on the

[1] In Ref. [6], however, it was found that the condition for the correct Newtonian limit is very marginally satisfied for the simple $1/R$ gravity. Thus, it may be likely that the deviation from the Newtonian gravity is observed even in vacuum solutions.

boundary of the gravitating body but, as we mentioned above, it tends to zero at very small distances from the boundary.

Let us consider now the internal solution with time dependent matter density. The first order correction to the curvature, $R = -T + R_1$ satisfies the equation:

$$\begin{aligned} \ddot{R}_1 &- \Delta R_1 - \frac{6\dot{T}}{T}\,\dot{R}_1 + \frac{6\partial_j T}{T}\,\partial_j R_1 + R_1\left[T + 3\,\frac{\dot{T}^2 - (\partial_j T)^2}{T^2} - \frac{T^3}{6\mu^4}\right] \\ &= D^2 T + \frac{T^2}{2} - \frac{3D_\alpha T\, D^\alpha T}{T}\,, \end{aligned} \tag{5}$$

where the deviation of the metric from the Minkowski one may be accounted for in the r.h.s. by the covariant derivatives (but this is not essential for our conclusion).

The last term in the l.h.s. has a huge coefficient:

$$T^3/6\mu^4 \sim (10^{-26} k^2 \mathrm{sec})^{-2}\left(\frac{\rho_m}{\mathrm{g\ cm^{-3}}}\right)^3, \tag{6}$$

where ρ_m is the mass density of the gravitating body, the constant k is assumed to be of the order of unity, $\mu = k/t_u$ and $t_u \approx 3\cdot 10^{17}$ sec is the cosmological time scale. This coefficient is much larger than T since

$$T \sim (10^3 \mathrm{sec})^{-2}\left(\frac{\rho_m}{\mathrm{g\ cm^{-3}}}\right). \tag{7}$$

Thus, the $T^3/6\mu^4$ dominates the coefficient in front of R_1 term in Eq. (6) and leads to strong instability. (Notice that $-T^3/6\mu^4 < 0$.) The characteristic time of the instability rise is about 10^{-26} sec. Perturbation with the wave length larger than one tenth of the proton Compton wave length should be unstable. This result shows that the curvature inside matter sphere should quickly rise and reach very high values (strong gravity) till some non-perturbative effects would terminate this rise. This is the same kind of instability that leads to the universe acceleration but now it is very unfavorable for the model and forbids such a modification of gravity.

It was argued in a recent work [7] that the considered above modification of gravity follows from certain compactification schemes of string/M-theory. If so, such theories must be excluded because of the instability found in the present paper.

The analysis performed above is valid for a particular case of $1/R$ terms in the action integral. A more general Lagrangian with arbitrary powers of R, both negative and positive, is presented in the recent work [8]. We have not made complete analysis of all possible cases discussed in the literature but it seems suggestive that all modifications of gravity at small curvature described by higher order differential equations may suffer from serious instability problems.

There are other earlier works on modifications of gravity [9]-[13] which might lead to accelerated cosmological expansion. However it is impossible to apply the similar analysis to some of them because the theory is not sufficiently specified - only a modification of the Friedmann equations is suggested but no modified action which would allow to derive equations in general case was presented [9]-[11]. Another group of works is based on modification of gravity due to higher dimensions [12]-[13]. Probably these models do not manifest instability discussed here, though only cosmological solutions have been considered in the quoted papers. In ref. [14] it is argued that modification of gravity at large scales to induce cosmic acceleration can lead to a modification of gravitational interactions at smaller scales but since the authors did not specify their model it is hard to make any definitive conclusions.

To summarize, we have shown that an attempt to explain the observed cosmic acceleration by the concrete modification of gravitational action adding to the latter the term $\sim 1/R$ is unacceptable because such term would lead to a strong temporal instability resulting in a dramatic change of gravitational fields of any gravitating bodies. We cannot make any conclusion about some other versions of the modified gravity theory which could lead to cosmic acceleration because the latter are not sufficiently well defined to permit such an analysis.

Acknowledgment A.D. Dolgov is grateful to the Research Center for the Early Universe of the University of Tokyo for the hospitality during the time when this work was done.

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